Robust controller and pre-filter design using QFT and interval constraint techniques

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Robust control system design for uncertain systems (aka “plants”) is of great practical interest, and their automation is a key concern in the control community [1]. In the last few decades, several robust control methodologies in time as well as frequency domains have been proposed (cf. [1]). Quantitative Feedback Theory (QFT) [2] is a one such frequency-domain controller design technique based on the use of Nichols chart [3] in order to achieve a desired robust design over a specified region of uncertainty in the plant parameters.

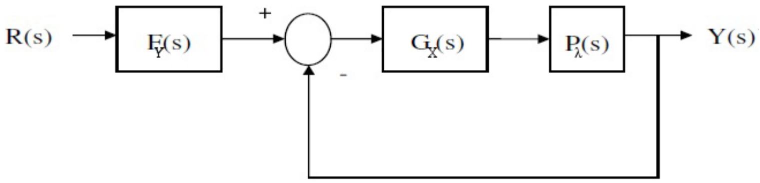

Fig. 1. Two degree-of-freedom structure used in QFT.

In QFT, the two-degree-of-freedom feedback system configuration is typically considered as shown in Figure 1, where $G_X(s)$ and $F_Y(s)$ are the controller and the pre-filter transfer functions, respectively parameterized by a set X and a set Y of design parameters. The uncertain linear time-invariant plant is given by $P_\lambda(s)$, where λ is a vector of uncertain plant parameters, real-valued scalars whose values may vary over a domain $\boldsymbol{\lambda}$, usually represented as an interval box.

The goal of robust controller and pre-filter synthesis is to identify values for the controller and pre-filter design parameters X and Y that satisfy a design specification in a robust way. For instance, the controller must ensure the stability of the close-loop system:

$$\forall \omega \in \boldsymbol{\omega},\ \forall \lambda \in \boldsymbol{\lambda},\ \left| \frac{G_X(j\omega)\, P_\lambda(j\omega)}{1 + G_X(j\omega)\, P_\lambda(j\omega)} \right| \leq \omega_s, \tag{1}$$

where the quantity $s = j\omega$ is substituted for the frequency domain (cf. [1]), $\boldsymbol{\omega}$ is an interval containing design frequencies ω, and ω_s is the robust stability margin specification. Similarly, the controller and pre-filter together must follow a desired performance specification. The lower tracking function $T_l(j\omega)$ and the upper tracking function $T_u(j\omega)$ are chosen based on the rise time and overshoot specifications.

$$\forall \omega \in \boldsymbol{\omega},\ \forall \lambda \in \boldsymbol{\lambda},\ |T_l(j\omega)| \leq \left|\frac{F_Y(j\omega)\,G_X(j\omega)\,P_\lambda(j\omega)}{1+G_X(j\omega)\,P_\lambda(j\omega)}\right| \leq |T_u(j\omega)|\,. \tag{2}$$

The quantifiers are usually eliminated from these constraints by considering a sample of design frequencies and a sample of uncertain plant parameters.

In addition, performance criterion are often considered to select the most appropriate controller or pre-filter amongst the ones satisfying the design constraints, turning the satisfaction problem into an optimization problem.

Originally the controllers and pre-filters have been designed manually, relying on designers skill and experience. However, the manual approach is often tedious and time taking, and usually leads to considerable "overdesigns" (see, for example, [4], and references therein): either the constraints are transformed into more restrictive ones that are simpler to express, or the performance criteria are ignored in the search of a feasible design. Motivated by these concerns, (semi)automated procedures have been recently proposed, derived from global optimization [6], interval analysis [7] or interval constraint satisfaction techniques (ICST) [5], [8].

Due to the challenging nature of the considered problem, the most recent approach [8] splits the problem in two: first, a controller is obtained considering a relaxation of the whole problem that eliminates the pre-filter influence from the considered constraints; second, a pre-filter compatible with the selected controller is obtained considering again all the constraints but fixing the controller parameters to the selected values. ICST have been successfully used for obtaining good controllers and prefilters for complex uncertain plants like the industrial plant emulator [5] and the magnetic levitation plant [8] depicted in Fig. 2.

Fig. 2. Schematic of the laboratory industrial plant emulator and magnetic levitation plant.

This approach, however, raises some issues when optimization is considered, because if the problem in the first step is relaxed, an optimal controller in this step may not be compatible with any pre-filter in the second step. In our talk, we will illustrate the current approach and its limitations. We will also discuss the possibility of addressing this challenging problem as a whole using ICST-based optimization techniques, and illustrate the obtained results on several test problems.

Acknowledgments

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